International
Mathematical
Union

IMU Fields Medal 2022
Maryna Viazovska Interview

Conducted by Andrei Okounkov & Andrei Konyaev

Part of the interview was recorded before February 24, 2022, another part — after. In both cases, the interviewers were Russian mathematicians

June 6, 2022

There is a war going on right now, the war to destroy my home country, my nation. This is done by the country of which you are citizens. People from your country are either doing this or supporting these actions, while assuring the world that there is nothing horrible about this. That a war is something that is going on in so many places around the world, that it has always been like this, that people have been destroying each other for millions of years.

Maybe this is true, maybe Homo Sapiens destroyed all other types of humans that existed in the past. And maybe war is indeed something inherent to humanity as a species. Perhaps anthropologists can justify this from a scientific point of view. But from a human point of view, killing and destroying an entire nation is not something normal.

I know people in Moscow, educated and well-read people, who at the same time support everything that is going on at the moment, everything that Russia does. Some of these people even go to church. Unfortunately, neither education nor profession can prevent people from turning into cannibals...

I realize that all of this is happening 3,000 kilometers away from me, I don't see bombs flying at me, children are safe and healthy. No one will break into my place and "denazify" me. But I want the people in Ukraine to know that they have our support. That I feel solidarity with them and it's not only me who feels that way.

****

For me, mathematics and strong emotions are incompatible. When the war started, at first I couldn't do anything at all. Now I have the sense that something must be done. I read the news about a professor from Uzhgorod University, who gives lectures by Zoom directly from the trenches. This story has inspired me a lot.

If we talk about what can and should be done, then we need to talk about the plight of the refugees. The crisis created a huge humanitarian problem and people in the world (including governments, organizations, and usual citizens) are working hard to deal with it. I am grateful to everyone who supports Ukraine. Let me speak only about one aspect close to my personal experience -- the plight of refugees and the effect of war on Ukrainian education. For example, Kyiv did not suffer as much as cities in the East, but 25% of students left Kyiv University. A huge number of children from Ukraine have now left for Europe, and they have to adapt to a completely different education system in a different language. And if school education is free almost everywhere, the situation with university students is more difficult –- it is hard for them to find a place in a European university. It is especially hard for those who have just graduated from school, or who are in their first year of university. In Switzerland, for example, graduates of a Ukrainian school have to study for one or two more years — this is both hard and complicated. The Swiss education system is doing a huge job to help Ukrainian children. However, it is impossible to replace what is lost and we will suffer the consequences of this war for generations.

I would like to thank everyone who helps refugees. Especially now, when the initial rally of support may be gradually fading away. It is clear that no one has expected things to grow to such a scale. And since Russia is continuing its military aggression, one should not expect the situation to improve in the near future.

February 18, 2022

How did you know that you want to become a mathematician? Does it run in the family?

We have no mathematicians in the family; my mom, dad, grandma, and grandpa are all chemists. It seems to me that my story is quite common. When I was 12 years old, I entered a school specializing in physics and mathematics and started taking part in mathematics olympiads. Then I realized that I wanted to be a mathematician.

Do you really mean you were devoted to math from that time on? No "crises of faith"?

There certainly were. The first occurred in the 11th grade when I didn't qualify for the International Mathematics Olympiad and felt discouraged from becoming a mathematician. That was a huge disappointment.

Then, at the university, I began to participate in student mathematics olympiads, to make up for the missed opportunities with the school olympiads. Probably, the next crisis happened when those student olympiads came to an end — because I became too old for them.

But then, luckily, I realized that there is such a thing as research in mathematics, where one can solve really hard problems and write papers about them. It was probably in my fourth year of university.

Do you have a favorite olympiad problem?

Well, I don't even know. Maybe that's why I wasn't totally successful in the olympiads. When you solve a lot of problems, you are more likely to remember the underlying principles. In any country, a finite number of people generate these contest problems, and if you devote yourself to solving everything they come up with, you can probably hack the system.

Of course, we trained for competitions, but not to the point of it becoming a drill; it was always a part of something bigger. We had an amazing physics teacher — who is still alive, by the way, 80 years old, still working at the school and still running a physics circle. The circle met on Saturdays. His philosophy has always been that you should be able to think, and not just crack olympiad problems. Because eventually, all olympiads will be over. And so he prepared us for that “after”.

Did you have a favorite kind of problems back then? Plane geometry[1], for example?

Actually, I disliked plane geometry problems. They got so very, very complicated at some point, and it was not clear exactly why they needed to be so complicated. It seems to me that there are a number of natural problems in plane geometry, but they run out quite quickly when it comes to olympiads. And so it starts like: "Let's consider 20 different circles, draw 30 lines, put 3 points on each line ..." and so on. I think we had one mathematician who would come up with such problems just for the Ukrainian olympiads.

That said, if you ask me what I do in mathematics, I'm likely to answer that it's geometry. Meaning, I love geometry. But these problems seemed very artificial to me.

Which olympiad or other math books did you like the best?

I loved those little books, you know, the Kvant Library and the like. There were several series of them. I remember having a book in which the fundamental theorem of algebra was proved by topological methods. Among the olympiad problems, I liked the combinatorics ones the best, especially the older ones. They may require just one idea, but a very beautiful one. In the newer ones, you may have to stack several ideas together — as if you were making a sandwich.

I also remember — this may not really have anything to do with math — we had a neighbor who died a long time ago. He was an old man who had fought in the war and then worked with my grandfather at the same university.

And he had such a huge — I mean, huge! — collection of different popular books on physics and mathematics at home. And then at some point, I think his grandchildren didn't get into science, and he gave them to me. The whole collection, the whole pile. And there, I found an astronomy book that really impressed me.

It was structured in a very compelling way, focusing on the theories of stellar evolution — from the 1920s to the more or less modern ones, up to, I think, the 1980s. And there were stories about different theories in which everything fit together really well — the theory, calculations, observations, … And then they would find a new star. And the whole theory would go in the trash. A new idea would be needed, a new theory, you'd have to start all over again. Some years go by, this more complicated theory works, and then another discovery happens, forcing it to be thrown out as well. And there were, I think, five iterations of that.

This really impressed me, because this was so unlike what we were taught at school. The teacher would say: “Here is the theory, you should learn it, and it will work”. But it turns out you can invent new theories!

But that's not how it works in math, right?

Of course, there are rare cases when a theorem is proved, the proof is accepted, and a few years later an error is found. But this is not what I mean. Times change, discoveries are made, maybe

[1] планіметрія

some previously overlooked aspects become important, maybe new ideas come from physics or astronomy. There is always room for a new theory in mathematics.

So, back to the fourth year at the university. The olympiads were over, you started doing research. Was it a long search before you came to geometry?

Well, it's me who thinks that I do geometry; maybe the geometers have a different opinion on this. Officially, I work in number theory. Anyway, it doesn't matter; I do a little bit of everything. Indeed, there was a certain period of search. I didn't start doing what I am doing right away.

When I was studying in Kyiv, I led a sort of a double life. Formally, I chose to specialize in algebra and entered the Algebra Chair[2], but I continued to be close friends with people at the Chair of Mathematical Analysis. We did projects together, wrote articles.

When I was in graduate school, I took up computer algebra. It seemed to me that the "computer" part of it was a very good thing because if I couldn't get a job as a mathematician, I would at least become a programmer. Eventually I wrote a program that counts certain curve invariants and realized that no, I definitely don't have plan B, I don't want to be a programmer.

It was by chance that I got to study with Don Zagier. I think that if I'm doing number theory, it's number theory in Don's sense. What I like about this part of mathematics is how it touches so many other areas. It's not a "thing in itself" when people create a theory within which everything is beautiful and they don't need anyone else. No, here you have an interface that connects you with almost everything you can think of — algebraic geometry, mathematical physics, analysis, and geometry.

So you were guided by Don?

Well, I don't know. It wasn't that he guided me; rather, he set an example for me. Don is very charismatic. If he has an idea, he comes running to the student's office to share that idea. If a student is lucky enough to be in the office at that time, they'll listen to Don for two hours. Usually, those two hours don't make any sense at all, but after two months there is this "Oh, this is it, this is what he was really telling me!" moment.

I had exactly the same story. I think Don was writing an article about Jacobi forms at the time. It's a long article joint with mathematical physicists, in which these modular forms appear as partition functions.

And so he realized an important thing about them. While the original function may seem to have no good properties whatsoever, it can be decomposed into a sum of three, and each summand has a good property. And each summand has a different one. For example, one summand is modular, but not holomorphic. And the second is holomorphic — but not modular. And so on.

---

[2] In many European countries, Mathematics Departments are further subdivided into Chairs (кафедра or катедра in Ukrainian), which group professors and students, both graduate and undergraduate, specializing in Analysis, Algebra, Geometry, etc.

And so he hit upon the idea of how to calculate it all, and he was telling me all about it. I don't remember that story in every detail, of course, but I remember the idea. That is, if there is an object that is bad in itself, it can be decomposed into a sum of good, understandable objects. But each one has to be good in its own way. And that this is how problems can be solved.

And what was the first research problem you solved?

Well, not many people paid attention to my first result. That was in the fourth year of university. This problem was suggested by Andriy Bondarenko, who was working with Andriy Prymak on rational approximation.

There is Bernstein inequality for polynomials, which implies that a polynomial bounded on the interval [-1,1] will have a bounded derivative at zero. This derivative is bounded by a constant which depends on the degree. When we approximate something by polynomials, this is a very important result, which can be used in many ways.

There is no such thing for rational functions. One can find a rational function of bounded degree which is bounded on the whole line, and yet its derivative at zero can still be as large as we want. Bondarenko and Pribak noticed that if you additionally require the monotonicity of a rational function then you can probably prove a Bernstein-type estimate. And this is exactly my first result — Andrii and I were able to prove this.

Do you remember how you proved it?

Yes! At the time, we had repairs going on in the kitchen. It was impossible to live at home, so my sisters and I moved to my grandmother's house. And my grandmother, of course, had her own rules; she is a strict person. At home, you could leave your cup of tea or socks in different places, but there it was crowded, not so much freedom.

At grandma's, everything was according to the schedule: we got up, had breakfast, and so on, there was cleanup, evening news at 9 p.m., and bedtime at 10 p.m. I had exams coming up and my grandmother made sure that I was preparing for them. I was so reluctant to study for the exams that I decided I'd rather think about the rational function problem. Grandma had no way of knowing whether I was preparing for the exams or solving a problem.

So I thought of a strategy to get an estimate. And then Andrii figured out how to make this estimate optimal, in the sense that it could not be improved. We wrote an article, but as it turns out, people are not very interested in monotonic rational functions.

Maryna, how do you choose problems in general?

I rarely start on new problems. I usually live with old ones for a long time and think about them. Well, how do I choose, I don't know. Of course, it has to be something interesting.

No — first, they must be interesting to me, and second, it should feel like I have the right tools to solve them. It may be a beautiful problem, but it may be meant for someone else and I have nothing to approach it with.

Tell us about the solution of the packing problem?

That was a little bit like the sandwich we talked about earlier. It took several steps to reach the solution. The first one, and it gave me the confidence that I will solve it, was when I managed to reduce the problem to a functional equation. I was coming home from a conference in Bonn. It was summer, it was rather stuffy on the train. On the train, I thought, since nothing seems to work, let me write the problem out one more time. In school, they taught us that your head is full of rubbish until you write things down to put them in order. So, I am writing it out, and I get this functional equation. I look at it and I think: "I should be able to solve it". And, indeed, I solved it, it only took a couple of months.

More precisely, there were two equations. The first one I solved very quickly, but the second one took two months. I remember how I came home to my parents in the summer, and wrote long, long formulas on pieces of paper in the evening. And so it happened that one of these formulas was the solution. Obviously, I made every possible mistake in these notes. But the final time I wrote it, I didn't make any mistakes, and there was the solution.

In hindsight, this functional equation could have easily been unsolvable. Had it turned out a little differently, it would not be solvable in terms of modular forms. So I was right to have my doubts until the very moment I found the solution.

An obvious question — how did you feel when you found out about the medal?

I don't know … Of course, I realize that this is such a unique thing, and I am so exceptionally lucky. And then I thought — how come? What am I going to do for the rest of my life? I'm just beginning to live, but I have already reached the highest point. I didn't like the idea of that at all. Then, of course, I also thought about how this imposes a great amount of responsibility on a person. It took me a few days to realize it all.

Now more students will come to you. You can give them mathematical problems to solve.

I don't know, it's kind of hard to give problems to students. Most of the time I give them topics to learn. In Lausanne, if you give your student a problem that they won't solve by the end of the semester, they may get upset. The students here are diligent and responsible but they need results.

I think that, back in my day, students were more carefree[3]. On the one hand, we were less organized, we could skip lectures. On the other hand, when I was a student, my teacher would say to me, "Well, solve this problem." I'd try to solve it for six months and just wouldn't be able to do it. And he'd say to me, "Well, you didn't succeed but you tried, good for you." However, there would be no result.

---

[3] розгильдяї

Now, as a Fields Medalist, you can influence mathematics education. What would you say, for example, to the Swiss Minister of Education about mathematics education?

My son is in school now, and at some point, I'd probably be interested in meeting with the people in charge of the math curriculum.

It always surprises me — if you look at the math curriculum, it changes all the time. When my son started learning geometry, I bought the textbook that I used. Written by Pogorelov back in the 60s. It's a great textbook, why would you need another one?

I understand that to say such a thing to the people involved in education would hardly be effective. There would probably be a lack of understanding between them and me. But it seems to me that this is a case when old is good. The plane geometry taught at school, it is still the same plane geometry as invented by Euclid, nothing has changed. We are already at a local, maybe even a global maximum. A maximum is beautiful, why change it?

I realize that I am different from other people in terms of math education. Maybe I don't have the same needs as others. And it is obvious that the textbook must be understandable to everyone, and accessible to all so that everyone can get the most out of it. But with the old textbooks, I like that there are really a lot of words and explanations in them. There are definitions, there are theorems. There is an explanation of what is a definition and what is a theorem. Modern textbooks are just an outline, a cheat sheet. Lots of pictures, not much text. Literally like in children's books — some small text in a large frame.

Perhaps one expects the teacher in class to fill in the voids, but it seems to me that the children in general rarely remember what the adults tell them — at least they are quite far from remembering everything. And having a book that says it all is actually a very good thing.

They used to teach mathematics in school in cycles: they revisited topics on different levels and it grew like a building — each floor came on top of the previous one. And modern education doesn't have that; all it has is snippets, bits, and pieces.

Perhaps people who work in education are aware of this. Maybe there is a contradiction between pedagogy and didactics. The way I see it, pedagogy constantly has to change, because every new generation is different from the previous one, simply because our lives are changing. And it seems logical to me that there must be some new approaches to communication between the teacher and the student — there should be. But the didactic part, consisting of theorems, proofs, with its strict logical structure — it makes no sense to change that. It was good for thousands of years; why break it down?

To be fair, I would like to say that there are many things I like about modern school and in particular school Switzerland. It is much more friendly to kids than what I remember from my own experience. A lot of emphasis is made on building good relationships between students, resolving conflicts, and collaborating on common projects. After all, school is not only about academic excellence. It is also about learning to live with other people.

What worries me sometimes is the following. There is a sense that people believe we do not need logic and knowledge anymore. There's not enough time for this. Machine learning will do everything for us instead, and we'll just look into a crystal ball and ask questions.

Do you often get questions on whether mathematicians will soon be replaced by computers?

I am still of the opinion that a computer is a tool. Meaning that computers don't get in the way of people — it's people who get in the way of themselves by using this tool. If you give a man a saw, he can cut firewood quickly, or he can cut off his own finger. If he saws off his finger, it's not the saw's fault.

The same thing applies to artificial intelligence. On the one hand, it certainly holds a variety of possibilities for mathematics and can complete many tasks which we would not be able to complete ourselves. The value of mathematics, on the other hand, is in being able to understand what's going on. To give meaningful tasks to the artificial intelligence and understand the answers. It's not like the computer will somehow think for me. Some people may be fascinated by the illusion of computer thinking, but it's just a tool.

But isn't our brain just a large neural network?

Indeed, but it is *our* brain! I am a humanist, for me people are special because people are us.

Do you use computers a lot?

In my work, I do a fair amount of numerical experiments. The computer is a very important additional tool. Unfortunately, I'm not that talented a programmer, so I usually need help from someone who can do it better. But there are things that are difficult to understand theoretically, and it's sometimes easier to test them by experiment. If the answer is negative, then all the constructions were wrong.

Also both in the sphere packing paper and the optimal energy paper, we use a computer to prove the positivity of a certain function. The function is rather explicit but very complicated, and to prove that it is positive directly would be very labor-intensive. Using interval arithmetic, a computer can check the positivity. So, a computer can provide not only inspiration, food for thought, but it can also certify proofs. In this sense, computers can prove theorems. But it would still be very nice to have a human who understands what is going on.

What do you do in your spare time?

I started running a couple of years ago, and I just love it. I tried to start drawing, but I think drawing is a bad hobby for a mathematician. Because you're still sitting there, you're still thinking about something. Too similar to mathematics.

I started running when I moved to Switzerland — my husband got me into it. And this, by the way, is a great hobby for a mathematician. It really does relieve the brain — I can't run and think. I don't know; there's probably not enough oxygen for everything. And it can be very useful for interrupting some vicious circle of thought if it has suddenly formed in my head.

I don't have any special accomplishments in running, but I try to run two or three times a week. I've recently participated in the Lausanne Marathon (I ran 10 km), which was virtual during the pandemic. When you have some time, you run around the lake, record a track on your phone, and upload it. They sent me a number and a T-shirt, which was a nice souvenir.